TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

2081 Chaitra

Exam.	**Regular (New Course)**		
Level	BE	**Full Marks**	60
Programme	All(Except BAR)	**Pass Marks**	24
Year / Part	II / I	**Time**	3 hrs.

Subject: - Engineering Mathematics III *(ENSH 201)*

- ✓ *Candidates are required to give their answers in their own words as far as practicable.*
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) Find the half range Fourier sine series of the function $f(x) = x$ in the interval $(0,2)$. [2]

   b) Find the Fourier cosine integral of $f(x) = e^{-ax}$ for $x > 0$, $a > 0$. [2]

   c) Find the Fourier transform of $f(x) = \begin{cases} 1 \text{ if } |x| < 1 \\ 0 \text{ if } |x| > 1 \end{cases}$ [2]

2. a) For a complex variable , show that $f(z) = \sin z$ is analytic. [2]

   b) Find the image of the circle $|z - i| = 1$ under the mapping $w = \frac{1}{z}$. [2]

   c) Show that $\int_C \frac{1}{z-a}\, dz = 2\pi i$ where C is a unit circle $|z - a| = 1$. [2]

3. a) Expand the function $f(z) = \frac{1}{z}$ about $z = i$ in Taylor's series. [2]

   b) Find the residue of $f(z) = \frac{3z-4}{(z-2)(z-3)}$ at each of the singularities. [2]

4. a) Solve the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 12x^2(t+1)$ subject to $u(0,t) = \cos 2t$ and $u_x(0,t) = \sin t$ [2]

   b) Find the solution of the equation $3u_x + 2u_y = 0$, by the method of the separation of the variables. [2]

5. a) If $Z[x(k)] = X(z)$, show that $x(1) = \lim_{z\to\infty} z[X(z) - x(0)]$, where the symbols have their usual meaning. [2]

   b) If $Z[x(k)] = X(z)$ and $y(k) = \sum_{m=0}^{k} x(m)$, then show that $Y(z) = \frac{z}{z-1} X(z)$. [2]

6. Find the Fourier series for $f(x) = x^2$ in the interval $-\pi < x < \pi$ and deduce the relation. [4]

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$

7. Find the Fourier integral of $f(x) = \begin{cases} e^{-x}; x > 0 \\ 0; x < 0 \end{cases}$ and hence show that [3+1]

$$\int_0^\infty \frac{\cos \omega x + \omega \sin \omega x}{1+\omega^2}\, d\omega = \begin{cases} \pi e^{-x} & ; x > 0 \\ \frac{\pi}{2} & ; x = 0 \\ 0 & ; x < 0 \end{cases}$$

8. Find the Fourier transform of $f(x) = e^{-ax}$ and use the result to show

$$\int_0^\infty \frac{dx}{(a^2+x^2)(b^2+x^2)} = \frac{\pi}{2ab(a+b)}$$ [2+2]

9. Prove that $u = y^3 - 3x^2y$ is harmonic function. Find its harmonic conjugate and hence construct corresponding analytic function $w = f(z)$. [2+2]

10. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$, using method of contour integration in the complex plane. [4]

OR

Evaluate $\oint_C \frac{\sin z}{z^2} dz$, where C is any closed path not passing through 0.

11. Apply Charpit's method to solve the PDE: $p^2x + q^2y = z$, where the symbols have their usual meanings. [4]

OR

Solve the partial differential equation $(y - z)p + (x - y)q = (z - x)$, using Lagrange's method.

12. Derive one dimensional wave equation representing the vibration of the particles a string. [4]

OR

Derive Navier-Stoke equation of fluid flow.

13. Solve one dimensional heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, subject to $u(0,t) = u(\ell,t) = 0$ and $u(x,0) = u_0 \sin\frac{\pi x}{\ell}$ [4]

14. Apply Z-transform to solve the difference equation $x(k+2) - 4x(k+1) - 5x(k) = 0$, given that $x(0) = 0, x(1) = 1$. [4]

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